# 13.1 — Space Curves

## What is a vector function?

$$\boldsymbol{r}(t) = \langle x(t), y(t), z(t) \rangle$$

*Equivalent*:

$$x = f(t), y = g(t), z = h(t).$$

**r**(t) is the position vector from the origin.

To visualize:

(1) find a containing surface/path;

(2) plot points and direction.

## 2D warm-up HW Q1

$$\boldsymbol{r}(t) = \langle -\cos(4t), t \rangle$$

Eliminate t to describe the curve.

- Where is the particle at t = 0?
- Which way does it move as t increases?

# Visualizing Curves in 3D

*Find the "surface of motion" first*

**Example 1**: $\boldsymbol{r}(t) = \langle 4\cos(t), 4\sin(t), 1\rangle$

- Use $\cos^2(t) + \sin^2(t) = 1$.
- What surface(s) contains the path?
- Plot points and mark direction.

**Example 2**: $x = t\cos(t), y = t, z = t\sin(t)$

- What surface is this on?
- What happens as t increases?

## Curve–Surface Intersections HW Q6–7

*Key to all intersections: Combine the conditions*

Substitute x(t), y(t), z(t) into the surface equation.

- Solve for t; substitute back to find points.

### Example 1

Where does $\boldsymbol{r}(t) = \langle t, 0, 3t - t^2 \rangle$ intersect $z = x^2 + y^2$?

### Example 2

Where does $\boldsymbol{r}(t) = \langle t, \cos(\pi\, t), \sin(\pi\, t) \rangle$ intersect $x^2 - y^2 - z^2 = 3$?

## Curve–Curve Intersections HW Q10–11

*Combine condition, but use DIFFERENT parameters*

$\mathbf{r}_1(t) = \mathbf{r}_2(s)$

- Solve for t and s, then check all three equations.
- Different parameter values can describe the same point.

### Example

Curve 1: $(x, y, z) = (t, 5t, 9)$

Curve 2: $(x, y, z) = (s - 2, 5, s^2)$

- Do the paths intersect? Where?
- Do the object "collide"?

## Surface–Surface Intersections HW Q8–9

*Key idea: Parametrize, and combine conditions if needed.*

Two surfaces typically intersect in a space curve.

**Many** different correct parameterizations are possible. If asked to find one example of a parameterization, we…

***Option 1***: Choose x = t, or y = t, or z = t, then solve for the others.

### Example

$z = 3x + y^2$ and $y = 5x^2$

- Choose x = t.
- Find y(t) and z(t).

# Surface–Surface Intersections: Circular Motion

**Option 2:** *When you recognize a circle*

$$x^2 + y^2 = R^2$$

use

$$x = R\cos(t), \qquad y = R\sin(t).$$

For an ellipse a similar idea works…

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

use

$$x = a\cos(t), \qquad y = b\sin(t).$$

**Example** *(from Math 224)*

Parameterize the intersection of

$x^2 + y^2 = 1$ and $z = 5 - x$.